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Stagnation-Point Heat Transfer to Surfaces of Arbitrary Catalycity

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Nomenclature

C = atom mass fraction
f = dimensionless stream function [Eq. (6)]
h = static enthalpy
h^0 = dissociation enthalpy
H = nondimensional enthalpy
k_w = wall catalycity
L = nose radius
M_2 = molecular weight of molecules
p = pressure
Pr = Prandtl number
q = heating-transfer rate
R = universal gas constant
T = temperature
u_1 = tangential velocity gradient [Eq. (6)]
U = freestream velocity
v = normal velocity
w = mass production rate
y = normal coordinate
η = transform coordinate
μ_s = viscosity at shock wave
ρ = density
ρ_s = density at shock wave

Introduction

To the authors' knowledge no solution has been published for the viscous shock-layer equations using an arbitrary chemical reaction rate and an arbitrary surface catalytic efficiency. Since a glide re-entry vehicle spends much of its time in a flight regime where nonequilibrium conditions are likely to be important, it is desirable to delineate this regime and be able to predict the magnitude of the heat-transfer reduction. Large reductions in heating rates can result in significant reductions in vehicle weights which, in turn, reduce the power requirements for launch and control of the vehicle.

Analysis

To study the effects of chemical nonequilibrium on heat transfer, a binary air model is used. To simplify the problem it is assumed that the constituents of the air model are in thermal equilibrium with the exception of the electronic states, which are assumed to be unexcited. It is further assumed that the Lewis number, Prandtl number, and the product $\rho\mu$ are constant, and the flight conditions are restricted to the regime where the effects of ionization on heat transfer are negligible. The parameters for the gas model are obtained from a weighted average of the properties of oxygen and nitrogen. In particular, the equilibrium constant for recombination and dissociation is calculated by averaging the characteristic temperatures of dissociation, vibration, rotation (as defined by Lighthill), and the individual partition functions for the ground state of the electronic mode. The final equations are those given in Ref. 1 and are repeated below:

Momentum

$$f''' + 2ff'' + \left(\frac{\rho_\infty}{\rho} - f'^2\right) = 0 \tag{1}$$

Energy

$$H'' + 2PrfH' = 0 \tag{2}$$

Species

$$C'' + 2PrfC' = -\frac{Pr}{(2)^{1/2}}\left(\frac{L}{U}\right)\frac{w}{\rho} \tag{3}$$

State

$$p = \rho(RT/M_2)(1 + C) \tag{4}$$

Caloric

$$h = \left(\frac{9}{2} + \frac{1}{2}\,C\right)\frac{RT}{M_2} + h^0C \tag{5}$$

where

$$\begin{aligned} \eta &= \left[\frac{2^{1/2}U}{\rho_s\mu_s L}\right]^{1/2} \int_0^y \rho dy \\ v &= -2^{3/2}\left(\frac{U}{L}\right)\frac{dy}{d\eta}f \\ u_1 &= 2^{1/2}\left(\frac{U}{L}\right)f' \end{aligned} \tag{6}$$

The boundary conditions for the foregoing equations are:

At $\eta = 0$

$$\begin{gathered} f_w = f_w' = 0 \qquad C_w' = \left(\frac{\rho_s\mu_s L}{2^{1/2}U}\right)\left(\frac{k}{\rho D}\right)_w C_w \\ H_w = \left(\frac{2}{U^2}\right)\left[\left(\frac{9}{2} + \frac{1}{2}\,C_w\right)\frac{RT}{M_2} + h^0C_w\right] \end{gathered} \tag{7}$$

At $\eta = \eta_s$

$$\begin{gathered} f_s = 2^{-5/4}\left(\frac{UL}{\rho_s\mu_s}\right)^{1/2}\rho_\infty \qquad f_s' = 2^{-1/2} \\ H_s = 1 \qquad C_s = 0 \end{gathered} \tag{8}$$

The properties behind the shock wave are obtained by solving the Rankine-Hugoniot equations using the hypersonic flow simplifications and assuming that the atom concentration is zero immediately behind the shock wave.

Results

Some results for a nose radius of 1 ft are shown in Figs. 1–3. Since it is not possible to include all the results in this note, the interested reader is referred to Ref. 2. Figure 1 shows the variation of heat transfer as a function of the freestream velocity at a fixed altitude of 230,000 ft and wall catalycities of zero and infinity. At a velocity of 15,000 fps

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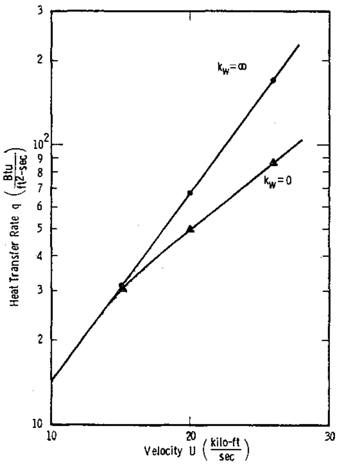

Fig. 1 Heat-transfer rate as a function of velocity for $k_w = 0$ and $k_w = \infty$ at 230,000 ft-alt.

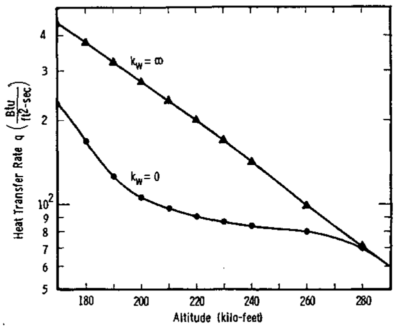

Fig. 2 Heat-transfer rate as a function of altitude for $k_w = 0$ and $k_w = \infty$ and a flight velocity of 26,000 fps.

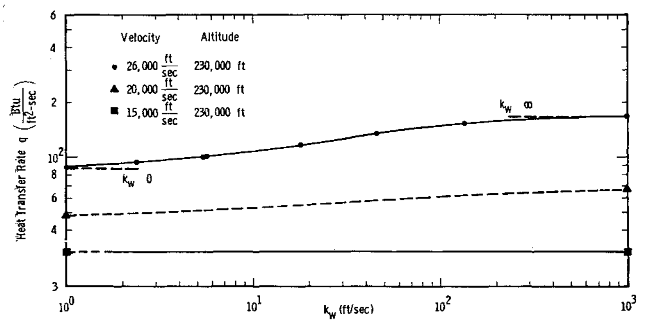

Fig. 3 Heat transfer as a function of wall catalycity.

the dissociation level is so low that the effect of wall catalycity is negligible and the gas behaves almost like an ideal gas. Further decreases in velocity will cause the flow to be frozen. Note that for the assumptions used in this analysis of shock-layer flows, the terminology "frozen" means that there is no dissociation in the layer. Figure 2 shows the change in heat transfer as a function of altitude for k_w equal to zero and infinity. At very high altitudes, the flow tends toward the frozen limit which will cause the curves to join; similarly, at very low altitudes the flow approaches equilibrium and again the curves will join. Figure 3 shows the effect of variations of wall catalycity on heat transfer for a fixed altitude of 230,000 ft and three freestream velocities.

For catalytic wall cases at low altitudes (below 200,000 ft) the heat-transfer predictions are in good agreement with the boundary-layer results of Ref. 3, whereas at higher altitudes, the results of the shock-layer calculations, due to viscosity in the shock layer, are greater than the results of Ref. 3. The scale law $\rho_\infty L$ = const was checked by comparing the solution for 230,000 ft and a 1-ft nose radius with the solution obtained for 265,000 ft and a nose radius of 5 ft. The resulting curves were virtually identical, verifying that this scaling law is accurate where dissociation is the predominant reaction.

A plot of the heating rate divided by the square root of the stagnation pressure (at constant velocity) as a function of the stagnation pressure or altitude shows that scaling according to the square root of the stagnation pressure is not valid for a noncatalytic wall when the flow is not in equilibrium. For the case of a catalytic wall, this method of scaling is poor when the flow approaches a frozen condition.

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A Simplified Formulation for Radiative Transfer

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Nomenclature

$B(y)$ = steradiancy, joule/cm^2
Bu = critical Bouguer number
$\mathbf{F}$ = flux of radiant energy, w/cm^2
F_y = radiant energy flux in the negative y direction, w/cm^2
h = physical thickness of the radiating slab, cm
k = linear absorption coefficient, cm^{-1}
N = conduction-radiation coupling parameter = $\lambda k/4\sigma T_{\max}{}^3$
θ = temperature ratio, $T/T_{\max}$; also, radiation angle of incidence
λ = coefficient of thermal conductivity, w/cm – °K
σ = Stephan-Boltzmann constant, 5.67×10^{-12} w/cm^2 – °K^4
τ = optical depth
τ^* = optical depth group = $\tau_i(1/\tau_0 + 1/\tau')^{-1}$
τ' = gradient optical depth = $T_{\max}/|\nabla T|$
ω = wave number, cm^{-1}

Subscripts

i = toward closest wall
max = condition at hot wall
0 = pertaining to whole slab expanse
P = Planck, referring to a gas that is optically thin
R = Rosseland, referring to a gas that is optically thick
ν = spectral

THE problem of radiative energy transfer in a hot flowing gas yields unwieldy integrodifferential equations that must be solved by numerical methods often requiring prohibitive computer time. The expense involved to acquire answers to questions of high-temperature radiation gas-dynamics is not justified because of the uncertainty in the values of the spectral absorption coefficients, k_ν. In spite of the difficulty of the calculations, Howe and Viegas[1] obtained numerical solutions to the radiating shock-layer case using the gray gas assumption. They were also limited to stagnation temperatures less than 15,000°K, and as a result could

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